Relative Identity and Relative Number

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1.   INTRODUCTION

Geach has claimed that Frege had an insight about number which should have led him to the doctrine of relative identity:

> I maintain it makes no sense to judge whether *x* and *y* are "the same" or whether *x* remains "the same" unless we add or understand some general term -the same *F.* That in accordance with which we thus judge as to the identity, I call a criterion of identity ... Frege sees clearly that "one" cannot significantly stand as a predicate of objects unless it is (at least understood as) attached to a general term; I am surprised he did not see that this holds for the closely allied expression, "the same." (Geach, 1962, p. 39)

> Frege has clearly explained that the predication of "one endowed with wisdom"... does not split up into predications of "one" and "endowed with wisdom." ...It is surprising that Frege should on the contrary have constantly assumed that "*x* is the same Q as y" does split up into "*x* is an *A* (an *y* is an t)" and "*x* is the same as...*y*" We have already by implication rejected this analysis (Ibid, pp. 151-52).

Here is the relevant passage from Frege, including his footnote:

> If it were correct to take "one man" in the same way as "wise man," we should expect to be able to use "one" also as a grammatical predicate, and to be able to say "Solon was one" just as much as "Solon was wise," It is true that "Solon was one" can actually occur, but not in a way to make it intelligible on its own in isolation. It may, for example, mean "Solon was a wise man," if "wise man" can be supplied from the context. In isolation,

however, it seems that "one" cannot be a predicate.* [*Usages do occur which appear to contradict this but if we look more closely we shall find that some general term has to be supplied, or else that "one" is not being used as a number word--that what is intended to assert is the character (not of being unique, but of being unitary).] This is even clearer if we take the plural. Whereas we can combine "Solon was wise" and "Thales was wise" into "Solon and Thales were wise," we cannot say "Solon and Thales were one." But it is hard to see why this should be impossible, if "one" were a property both of Solon and of Thales in the same way the "wise" is (Frege, 1884/1960, p. 40).[i]

Another passage, which it will be helpful to have before us, is this one:

A colour such as blue belongs to a surface independently of any choice of ours...The Number 1, on the other hand, or 100 or any other Number, cannot be said to belong to the pile of playing cards in its own right, but at most belong to it in view of the way we have chosen to regard it and even then not in such a way that we can simply assign the Number to it as a predicate (Ibid, p. 29).

I have defended Frege's failure to adopt the doctrine of relative identity in "The Same F".[ii] But I did not there defend the consistency of his doctrine of number and his treatment of identity. I wish to do so here.

2.   WHAT IS RELATIVE IDENTITY?

First, however, we must remind ourselves what the doctrine of relative identity is, and is not.

The doctrine of relative identity includes the claims that (i) "x is the same A as y" does not "split up" into predications of "x is an A (and y is an A)" and "x is the same as y"; (ii) There are, or could be, cases of "x is the same A as y, but x and y are different Bs," where A and B are countnouns (Geach, 1962, pp. 151-52); (iii) There is no such thing as being just "the same" (Geach, 1962, p. 157; Geach, 1972,

p. 249).   I take (i) to be the central claim, with which Frege clearly disagreed) (ii) is evidence for it, and (iii) a consequence of it.

The doctrine of relative identity can easily be mistaken for more plausible doctrines, with which there is no evidence that Frege would disagree, and which do not support claims (i), (ii), and (iii).

A) **The doctrine that singular terms are count-noun laden**. The function of singular terms is to identify entities and it's plausible to suppose that such identification requires an understanding of what kind of entity is being identified. If I extend my finger towards a building and say, "That is what Jones donated to the university" you may understand me to be making a cutting remark when my motive is laudatory. You take me to be referring to a brick, where I meant the whole building. The problem is with the singular term; if I had said "this building" I would have succeeded in identifying what I was talking about. This plausible doctrine does not support, but undercuts, the doctrine of relative identity. We might have thought this latter doctrine was supported by an example like the following: Heraclitus points at successive moments toward the Cayster, saying "This isn't the same as that." If we think "this" and "that" are singular terms in good order, we might think a good explanation of the indeterminate nature of what he's saying is that we don't know whether he has in mind river identity (in which case he's wrong), or collection of molecules-identity (in which case he's right). And this might lead us to think that "This is the same river as that, but this and that are different collections of water molecules" is good support for claim (ii) or relative identity, and hence for claim (i). But the singular terms are incomplete) the problem is not that we don't know "what kind of identity" is in question, but that we don't know whether Heraclitus is making a silly remark about a river or a substantial point about collections of water molecules.

B) **The doctrine that everything belongs to some kind or another.** This is perhaps a denial of one version of the doctrine of "bare particulars." Given any statement of the form $"x$ and y are the same" there will be a true "completion" of it

of the form *"x* and *y* are the same *A*."* Given the last doctrine, the completion may, of course, be redundant. These two doctrines guarantee that in such statements of identity there will be explicitly or implicitly understood some "count noun" or "sortal." The plausibility of these doctrines does not add to the plausibility of the doctrine of relative identity, but subtract from it, for they provide a less drastic explanation for the facts cited as evidence for that doctrine.

C) **The doctrine of the diversity of criteria of identity.** In judging that the man I saw last week is the one before me now, I do not use the same criteria I use in judging that the same battleship that was docked at Long Beach last Friday is still docked here now, or in judging that the number of Billie Jean King-rooters in my living room is the same as the number of people in my living room.

But what are criteria of identity? As Geach has observed, the word "criteria" obscures an important distinction between the kinds of evidence usually employed in making identity judgments of a certain kind, and the conditions of identity. For example, it's good evidence that the ship there last Friday is the one there now, that the registration number on the hull is the same. But sameness of registration number is neither necessary nor sufficient for ship identity. Presumably something like spatio-temporal continuity, which would seldom be directly observed by even the crew over a week-long period, is necessary and sufficient.

But on either understanding of "criterion of identity," it seems clear that the criteria of identity are relative to the kind of item in question. If we confuse our criteria of identity, in either sense, with the relation of identity, the doctrine of relative identity will follow straight away. But this is a mistake.

3.   F**REGE ON CRITERIA OF IDENTITY**

Frege was well aware of the fact that different kinds of objects have different criteria of identity. When he is trying to give a criterion of identity for numbers (in the second sense), he pauses, and fixes the criterion of identity for directions as an example. The criterion of identity for directions is parallelness, for numbers it

is equinumerosity (Frege, 1884/1960, p. 79).[iii] But parallelness and equinumerosity are not, so to speak, on the same level as identity. Parallelness is a relation between lines; equinumerosity a relation between concepts. In each case, when we give the criteria for the identity of As, we are not saying what relation As must be in to be identical, but saying what relation some other sorts of entities (lines or concepts, in this case) must be in to be instances of the same *A*. Equinumerosity and parallelness are not two kinds of identity, one for directions and one for numbers.

When Frege observes that

a is parallel to *b*

comes to the same thing as

the direction of *a* is identical with the direction of *b*

his point is to explain the meaning of "the direction of __." This needs explanation, he argues, for we have no intuition of directions, although we do have intuitions of straight lines and parallel lines. We understand already the concept of identity, for "in universal substitutability all the laws of identity are contained.' (Frege, 1884/1960, p. 79) Thus only directions remain to be understood, and the above equivalence almost succeeds:

We carve up the content in a way different from the original way, and this yields us a new concept. (Frege, 1884/1960, p. 75)[iv]

Frege's general conception of a criterion of identity seems to me to be sound, and susceptible to generalization beyond the abstract sorts of entities with which he was wont to deal.

For any kind of object *A*, we can ask (i) How do As manifest themselves? What entities play the role for As that straight lines play for directions? We can call this the class of *A*-occurrences; (ii) What relation plays the role for As that parallelness plays for directions? I.e., what relation obtains between *A*-

occurrences of the same *A*. Such a generalization of Frege's schema might allow for different occurrence relations, as well as different "criteria of identity," for objects of different kinds or categories.

This scheme needs much working out, no doubt. But it has at least one merit. It allows us to appreciate the diversity and importance of criteria of identity, while distinguishing this from the doctrine of relative identity.

4.   FREGE ON NUMBER

Now let us turn to Frege's view about number, to see whether it should have led him to the doctrine of relative identity.

The remarks in the second quote from Frege at the beginning of this paper occur as he is exploring the suggestion that number is "on a level with colour and shape...a property of things." Frege objects:

> If I place a pile of playing cards in [someone's] hands with the words: Find the Number of these, this does not tell him whether I wish to know the number of cards, or of complete packs of cards, or even say of honour cards at skat. I must add some further word -- cards, or packs, or honours. (Frege, 1884/1960, p. 28)

So far, then, Frege has denied that Number is a property of a pile of cards In the way that being blue might be a property of the pile. At this point, he might adopt what I shall call the doctrine of relative numbers: "having the number two" is not a single property. There is no such thing as having the number two simpliciter. There are just a bunch of relative number properties: having the pack-number two; having the card number two; having the honors-at-skat number two, and so forth. Once we see this, we are free to allow that the pile, after all, has the number properties. It has the pack-number two, and the card-number one hundred four. It does not, however, have the pack-number one hundred four, or the card-number two. There is no number it both has and has not, and no problem. The doctrine of relative numbers would be a reasonable stablemate for the doctrine of relative identity. To say that *x* is identical with *y* is to say that *x*

and *y* are one. So the need to ask, "which kind of identity," pressed by the doctrine of relative identity, is merely a special case of the need to ask "what kind of number," pressed by the doctrine of relative numbers. It seems clear that Frege did not adopt anything like the doctrine of relative numbers. Rather than multiplying the kinds of numbers attributed to the pile, he rejects the idea that the pile has a number at all. In the quote just given four paragraphs back, the following remark was omitted:

> To have given him the pile in his hands is not yet to have given him completely the object he is to investigate. (Frege, 1884/1960, p. 28)

> A little later he says, an object to which I can ascribe different numbers with equal right is not really what has a number. (Frege, 1884/1960, p. 29) In remarks quoted at the beginning of this paper, Frege seems to allow a sense in which numbers belong to the pile,

> The number l...cannot be said to belong to the pile of playing cards in its own right, but at most to belong to it in view of the way we have chosen to regard it... But this sense is pretty weak and even then not in such a way that we can simply assign the number to it as a predicate. (Frege, 1884/1960, p. 29)

> If number is not a property of the pile, what is it a property of?

A considerable portion of the Foundations of Arithmetic is occupied with that question. The solution comes in section 46:

> …the content of a statement of number is an assertion about a concept. If I say "Venus has 0 moons"...a property is assigned to the concept "moon of Venus," namely that of including nothing under it. If I say "the King's carriage is drawn by four horses," then I assign the number four to concept "horse that draws the King's carriage. (Frege, 1884/1960, p. 59)

This explains why we must "add some further words -- cards, or packs, or honours" before the question "Find the number of these" tells the recipient of the card pile what we want to know. These additional words don't tell him what kind

of numbers are involved, but which concept is being asked about. Thus when I say, as perhaps a philosopher might, pointing at a pile of cards,

>1)   The number of these is two.

I might have in mind either the true statement

>2) The number two belongs to the concept, packs contained in this pile.

or the false,

> 3) The number two belongs to the concept cards contained in this pile.

The difference between (2) and (3) is not the kind of numbers involved. The number two that belongs to the first concept is just the number two that doesn't belong to the second. The difference lies in the concepts to which that number is asserted to belong.

It is, then, incorrect to say that "Frege sees that "one" cannot significantly stand as a predicate of objects unless it is (or at least understood as) attached to a general term..." (Geach, 1962, p. 39) "One" is, according to Frege, not a predicate in any case, but the name of an object, the number one. (Frege, 1884/1960, pp. 67ff.) What gets predicated is "having the number one." And the general term (e.g., "card" or "pack") functions to identify the concept which is asserted to have the number. As Frege puts it:

> Several examples given earlier gave the false impression that different numbers may belong to the same thing. This is to be explained that we were there taking objects to be what has number. As soon as we restore possession to the rightful owner, the concept, numbers reveal themselves as no less mutually exclusive in their own sphere than colours are in theirs. (Frege, 1884/1960, p. 61)[v]

5.   A TENSION IN FREGE'S ACCOUNT?

But still, if Frege held that having the number one-is a property of concepts, shouldn't he have held this too of identity, given the intimate relation between identity and oneness? So isn't something awry with his account of identity?

The intimate relation between identity and oneness is illustrated by the fact that (4) and (5) come to the same thing.

(4) Flora and Bossie are the same.

(5) The number one belongs to the concept is Flora or is Bossie.

To obtain the identity predicate from (4), one would erase "Flora" and "Bossie, yielding (6):

(6) [ ] and ( ) are the same.

The same operation with (5) gives us (7):

(7) The number one belongs to the concept is [ ] or is ( ).

The fact that (6) and (7) are predicated of cows and not concepts in no way threatens Frege's claim that (8) is predicated of concepts:

(8) The number one belongs to { }.

The relation between the property of having the number one and the relation of identity is still intimate enough: the property of having the number one will belong to any nonempty concept all of whose instances are identical.

(4) and (5), of course, might fail to express a complete thought. Suppose, for example, a rancher in the habit of naming both his herds and his cows point in the direction of the same cow, but different herds, on successive days (the cow having changed pastures), saying, "That's Bossie" the first day and "That's Flora" the second. Then we might not know whether (4) and (5) said something true

about a cow with two names, or something false about herds. This might be cleared up by addition of the count-noun "cow":

>   (9) Flora and Bossie are the same cow.

>   (10) The number one belongs to the concept, cow that is Bossie or Flora.

In the case of (9), the addition of "cow" would clear things up by telling us what we are talking about. In (10) it would answer the question: of which concept is having the number one predicated? In a sense, (4) and (5) are incomplete for the same reason. We do not know of which concept having the number one is predicated in (5) because we don't know to what objects "Bossie" and "Flora" refer. The power of the word "cow" to clear up these questions requires no explanation by the doctrines of relative number and relative identity.

Thus I claim that Frege's views about number and identity in *Foundations of Arithmetic* are consistent. *A* question remains, however. For it is not at all clear that either of these views are consistent with the passage Geach actually cited, which I have not yet discussed.

6.   A TROUBLESOME PASSAGE

In this passage Frege says that if "one man" should be taken as analogous to "wise man," "one" should be a grammatical predicate. The suggestion seems to be that it is not. Although "Solon was one" actually does occur, it, unlike "Solon was wise," is not "intelligible on its own in isolation." It might mean "Solon was a wise man." The point Frege is making is supposed to be clinched by observing that "...we cannot say Solon and Thales were one."

Frege might mean to be making a point about identity and individuation here. He might be supposing, with regard to "Thales" and "Solon," that they are ambiguous in the way we imagined "Flora and Bossie" to be, that in addition to being names of different men, they are used as names of the same herd, or pack of men, or man-fusion, or man-aggregate, or committee. His point would then be that without a general term, like "man," which tells us what we are talking about,

we haven't said anything determinate. If this is what Frege is saying, this passage is consistent with, and supports, his views about number and identity. But I really don't think Frege has anything like this in mind. If he had, he would probably have said so, or at least used an example, like the pack of cards example, more appropriate to the point.

In the section in which this passage occurs, Frege is arguing against a view, which he finds in Euclid and Schroeder, that units are a certain kind of thing, those things with the property expressed by "is." In addition to his earlier arguments against the view that number is a property of things, he adds this one:

> It must strike us immediately as remarkable that every single thing should possess this property. It would be incomprehensible why we should still ascribe it expressly to a thing at all. It is only in virtue of the possibility of something not being wise that it makes sense to say "Solon is wise." The content of a concept diminishes as its extension increases; if its extension become all-embracing, its content must vanish altogether. It is not easy to imagine how language could have come to invent a word for a property which could not be of the slightest use for adding to the description of any object whatsoever.

The passage in question occurs immediately after this.

If we ask ourselves what Frege could be saying in the passage in question, which would support and is required by this argument, we are led to the following interpretation. "Solon is one" has a use only when there is a non-all-embracing concept in the background, for example, as an answer to the question "Were there any wise men in those days?" It does not tell us some property of Solon's expressed by "is one," for "is one" is only grammatically, and not logically, a predicate. It tells us how many, or at least how many, things have the background property; i.e., how many things fall under a certain concept. On Schroeder's view, since both Solon and Thales are units, it should follow that "Solon and Thales are one." But this does not follow, for it is not true.

Frege clearly thinks that if someone were to say "Solon was one" or "Solon and Thales were two," it's appropriate to ask "One what?" or "Two what?" In this way, these statements are similar to "This is the same as that" (same statue or same lump of coal?) or "There are two of these on the page?" (two types or two tokens?). But on this interpretation, the resemblance would be misleading. For the "One What?" question is not a request for a criterion of identity, to tell us what is being talked about (or in Geach's view, what kind of identity is involved), but rather a request for a non-all embracing concept, relative to which the claim will be of some interest.

Interpreted this way, Frege has said everything he needs to say about Schroeder, and has said nothing inconsistent, or even particularly relevant to, his views on number and identity.

But unfortunately, this doesn't seem to be what he is saying either. For he does not say that although "is one" can be a grammatical predicate, grammar is misleading here; he seems to say it just can't be a grammatical predicate. And he does not simply say that such sentences as

(11) Solon is one.

(12) Solon and Thales are two.

are, in the absence of a non-all-embracing background concept, useless, but that they are unintelligible. And he does not simply say simply that (12) is false, but that it's something "we cannot say."

What he does actually say seems to me inconsistent with his view of number, false, and unmotivated.

Consider

(13) Thales and Solon are two wise men.

(13) seems like it should be acceptable to Frege. But (13) seems to be an attribution of a number to something. On Frege's view, it would have to be

concept that has the number two. Which concept is it? Two is not the number of wise men, (or if it is, (13) doesn't tell us so). Two is the number that belongs to the concept, is Solon or is Thales. But then (13) must amount to

> (14)   Solon and Thales are wise men, and the number two belongs to *is Solon or is Thales.*

But if this makes sense, its second conjunct should make sense,

> (15)   The number two belongs to *is Solon or is Thales.*

But if (13) is the ordinary way of expressing the thought better expressed by (14), shouldn't the ordinary way of expressing the thought better expressed by (15) be (12)? But Frege seems to think that (12), except as an incomplete version of (13), is unintelligible. So Frege's view of number seems to lead to the conclusion that (12), however useless, is perfectly intelligible, while in the passage in question he denies this.

He seems wrong to deny it. Its dubious that (12) is even useless much less unintelligible. If someone thought that "Thales" was Solon's pseudonym, (12) would seem an acceptable way to tell him otherwise. It certainly seems to make sense. The utility of (11) seems less secure' But we might want to deny, say, "Ellery Queen is one," given that Ellery Queen's novels are co-authored. If so, "Agatha Christie is one" then seems true and useful. It seems to make perfectly good sense; and Frege's view of number seems to show us the sense it makes.

It's hard to see why Frege should have said things that aren't required for his argument, aren't true, and aren't consistent with the account of number which is his main accomplishment in the *Foundation of Arithmetic*. Perhaps he really was thinking about identity and individuation. Or perhaps he got careless.

Has Frege then clearly explained something in this passage, which had he followed it through, would have led him to the doctrine of relative identity?

I don't think so, because I don't think Frege explains in this passage anything clearly at all.

7. CONCLUSION

To sum up. The account of number Frege puts forward in the *Foundation of Arithmetic* is compatible with the account of identity he gives there. And he does not put forward the doctrine of relative number, which seems a natural extension of the doctrine of relative identity.

On the othe[vi]r hand he does say, in the passage Geach actually cites, something incompatible with the view about number he later develops.

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[i] . The *Grundlagen*, from which Geach's quotes and these quotes are drawn, was not Frege's major work, and not his last word on the issues here discussed. When I speak of "Frege's views," I mean only his views in the *Grundlagen* (1884/1960).

[ii] See also Wiggins, 1967; Nelson, 1970; Feldman, 1969. Geach replies to Feldman in 1969b, and seems to include other critics in *Logic Matters*, (1972). In correspondence, Geach has informed me that my criticisms are based on misunderstandings and not worth replying to in print. I am unconvinced, however, of any misunderstandings relevant to my criticisms of relative identity. I did, as Jack McIntosh has observed, take Geach to say "polluted" at one point where he said "pollulated." An excellent discussion of Geach's views on identity and related matters appears in Dummett, 1973, chapter 16. On the whole, Dummett does an excellent job separating the insightful from the implausible in Geach's writings on these issues. However, Dummett, and also W. V. Quine in (1973) maintain that something like Geach's doctrine of relative identity is true "as long as the sides of the identity sentence are demonstrative pronouns." (Quine, p. 59; see Dummett, pp. 570-95.) It is true that such sentences, and many others not involving demonstratives, are in some sense incomplete or deficient, and can be completed by inserting an appropriate general I term after the work "same." (See "The Same F, p. 184, and below, section A.) But the problem is not, as Dummett supposes, that in such sentences "it is correct to say, with Geach, that 'the same' is a fragmentary expression." (p. 570) For surely the indeterminateness of "This is the same as that," (said, e.g., by Heraclitus' wife, rather slowly, with two pointings towards the Cayster) has the same source as the indeterminateness of "This was here yesterday," (said in similar circumstances), in which the allegedly fragmentary expression does not occur. The information required, in both cases, to make good the indeterminancy, is which objects are referred to, not what kind of identity is predicated. (If we suppose, with Quine, that we are at a stage of language at which experience has not yet been clumped into objects, it is surely not the kind of identity that is in question. No identity, we might say, without entity. (Even if we don't say that, as I argue we shouldn't in the next essay, the issue seems to be how we individuate, not what kind of identity.)) In both cases, inserting the appropriate general term after the demonstrative will suffice. In spite of his admirable analysis of a wide variety of examples, I believe Wiggins is also not completely clear about this; his treatment seems to involve retaining different kinds of identity, while not allowing the possibility of "same F, different Gs," emphasized by Geach. See Shoemaker, 1970b Perry, 1970b.

[iii] Frege's word for the "possibility of correlating one to one the objects which fall under the one concept with those which fall under the other" is *gleichzahlig*, which Ausin translated "equal"; I prefer "equinumerous."

[iv] I say "almost," for Frege finds a difficulty: we haven't yet explained why England, for example, is not a direction. He solves this problem by defining the direction of line a as the extension of the concept, parallel to line *a.*.

[v] Further light on these issues has been shed by Jonathan Bennett and Willliam Alston (1984) and Patricia Blanchette (1999)….[A fairly long note, about half a printed page, will be inserted at this point -- JP]

[vi]