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Three Paradoxical Aspects of Identity. by Heinrich Behmann; Max Käsbauer; Franz von Kutschera; Verlag Karl Alber

Review by: John Perry

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of the abstract proofs at all, only the soundness of the principles (and not the order in which they are) used.

The conflict, just mentioned, between logical and mathematical requirements on proofs suggests a parallel in connection with idealized and feasible computations, contradicting somewhat page 105. Specifically, inasmuch as the problems of genuine feasibility, which correspond to mathematical requirements, are not refinements of (natural) problems about idealized computers, the latter are liable to be misleading. Again, at the top of page 107, the philosophical status of idealized computability seems to be also somewhat exaggerated (in the comparison with definability and provability). After all, computability is nothing else but a very special kind of definability, of predicates whose arguments are required to be presented finitistically and "processed" mechanically. Again, not general provability but the special kind determined by (f.o.) logical consequence seems to be a realistic analogue to (idealized) computability, and that special kind is no less absolute than computability. Of course these reservations about the practical and philosophical uses of idealized computations do not affect their permanent interest for the study of finitely generated groups and Diophantine equations.

GEORG KREISEL

HEINRICH BEHMANN. *Drei Aporien der Identität. Logik und Logikkalkül*, edited by Max Käsbauer and Franz von Kutschera, Verlag Karl Alber, Freiburg and Munich 1962, pp. 19–48. Reprinted in *Ratio* (Frankfurt a.M.), vol. 5 no. 2 (1963), pp. 101–123.

HEINRICH BEHMANN. *Three paradoxical aspects of identity*. English translation of the preceding. *Ratio* (Oxford), vol. 5 no. 2 (1963), pp. 113–139.

(1) Behmann observes that "The morning star is identical with the evening star" is a non-trivial, true, "empirical, synthetic judgment . . . obviously neither a 'tautology' nor a 'contradiction'" (in translation, p. 114; in original, p. 20). But as long as we analyze the statement on the "level of naive understanding," in terms of the thing denoted by the singular terms (Venus) and the relation of identity, we will not capture the difference between this statement and "Venus is identical with Venus," which *is* trivial. The solution is to analyze the sentence at the "level of critical understanding." At this level, we deal not with *things* (extensions or denotations), but objects which are *not* things (viz. intensions or contents). The latter level of analysis differentiates between the two statements, for at this level of analysis we find different intensions (the intensions appropriate for singular terms) associated with "the evening star" and "the morning star" even though their denotation is the same.

Explanations of the significance of some identity statements by reference to a difference in entities of some kind (intensions or senses) somehow associated with the singular terms in spite of their shared denotation were familiar from Frege (498), Church (XVII 133), and Carnap (*Meaning and necessity*, Chicago 1956). Behmann's discussion does not approach these in rigor or sophistication.

Original and interesting are Behmann's characterization of intensions as entities which are not identifiable and his choice of *indicates* as the appropriate intensions for singular terms.

According to Behmann, the proposition  $a = b$  will always be true or false if  $a$  and  $b$  "represent" identifiable objects, but not if they represent objects that are *not* identifiable. Examples of the latter are *propositions*. The question whether or not the proposition expressed by "Through two points there passes at most . . . one straight line" is identical with the proposition expressed by "Two straight lines have at most one common point" is not just difficult, but illegitimate (p. 116; p. 22). Behmann claims that intensions (propositions, relations, properties, and indicates) are distinguished by this criterion from extensions or things, which include "concretely identifiable objects," classes of these, numbers, and functions (p. 118; p. 24).

But even with the most concrete of objects, there are circumstances in which identity statements concerning them have no clear truth-value. For example, on Behmann's criterion, ocean waves, the identity conditions of which are notoriously obscure, would be intensions. Moreover, Behmann's criterion appears to give us mutually exclusive classes of intensions and extensions or (as he takes to be equivalent) denotations. But at least some intensions can be extensions or denotations, for intensions can be denoted. Behmann's awkward stipulation

that among the "things" will only be classes with things as elements is a symptom of this problem. The status of classes of intensions is left obscure.

Indicates apparently include individual concepts, but also what is "represented" by a pointing gesture. Indicates are not identified, for an indicate is not identifiable, not a "thing." Thus when I say "This is the same pencil as that" (not Behmann's example), although the identifiable object pointed to by both indications is the same (a certain pencil), the indicates are not. Behmann, citing Bolzano, argues that the content of statements cannot be captured "in a purely conceptual manner," and that "*the indicates* (as essentially prelinguistic entities) *will have to be taken into account within the system of formal logic and to be explicitly incorporated into it*" (p. 122; p. 30).

(2) Behmann next searches for "the condition under which, on the basis of an identity . . . a substitution is unobjectionably admissible" (pp. 125–126; p. 34). The solution is that substitutivity is legitimate if the proposition in which the "extensionally equal" singular terms or predicates are to be substituted are "extensional with respect to the constituent to be replaced" (p. 126; p. 34). Behmann provides a list of concepts which play a part in non-extensional propositions. Behmann does not distinguish, as Carnap had done (op. cit., section 13) between concepts which provide non-extensional propositions in which substitution of intensionally equivalent expressions is valid ("It is necessary that . . .") and those for which an even tighter condition seems to be required ("Smith doubts that . . ."); nor does he attempt to work out any systematic account of non-extensional propositions.

(3) Behmann maintains that on an "ontological" interpretation, wherein properties are restricted "to those which, when represented in words or symbols, do not explicitly refer to" the particular objects in question (p. 130; p. 38), the principle that indiscernibles are identical is not a necessary truth. Behmann cites as counterexamples objects in a radically symmetrical universe, and the imaginary numbers  $i$  and  $-i$ . On the basis of these counterexamples, Behmann rejects the *Principia mathematica* definition of identity as indiscernibility relative to predicative properties.

To the second counterexample, one might respond that on any particular set-theoretic construction of arithmetic,  $i$  and  $-i$  will be entities discernible by their set-theoretical properties in spite of indiscernibility with regard to arithmetic properties.

Behmann includes a brief discussion of the axiom of choice. He argues that while at a purely intensional level the axiom of choice is not a law of logic, it is at an extensional level. This reviewer has not been able to grasp the reasoning of this discussion.

JOHN PERRY

FRED SOMMERS. *On a Fregean dogma. Problems in the philosophy of mathematics, Proceedings of the International Colloquium in the Philosophy of Science, London, 1965*, volume 1, edited by Imre Lakatos, Studies in logic and the foundations of mathematics, North-Holland Publishing Company, Amsterdam 1967, pp. 47–62.

As an example let the predicate be mortality; and the predicate's contrary, therefore, immortality. Then  $A$  affirms the predicate, *Men are mortal*;  $E$  affirms the predicate's contrary and so contraffirms  $A$ , *Men are immortal*;  $I$  denies the predicate's contrary and so contradenies  $A$ , *Men aren't immortal*;  $O$  denies the predicate and so denies  $A$ , *Men aren't mortal*. (This may be arranged in the form of the traditional square of opposition, with  $A$ ,  $E$ ,  $I$ ,  $O$  at the corners.)

This reinterpretation of the classical categorical forms is Sommers's "predicative scheme." By its means the customary distinction between universals and particulars becomes one of affirmation versus denial (distinctions of quality are as well "absorbed"). On this trick of "dequantifying" the forms turns the motivation of Sommers's thesis: that quantificational logic, in treating predication as logically singular— $(x)(Fx \supset Gx)$  signifying that the  $F$ 's, taken individually, are each  $G$ 's—is guilty of the erroneous dogma of singular predication, and that as a consequence, the Fregean dogma (by which is meant that the ancients were mistaken in supposing *Socrates is mortal* and *All men are mortal* to be of the same form) is without foundation. Sommers holds that his predicative scheme "requires no distinctions of 'distribution.' The subject . . . remains the same in all four categoricals."

He readily acknowledges the "logical distinctions arising out of the difference between singular